Please use the attached FX time series data EURUSD for this assignment, and please send back your codes and the wrap-up of your results to us.

**Task 1: Calibration:**

Option 1: Assuming that the FX spot rate follows Geometric Brownian Motion:

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, calibrate the and by matching the 97.5th and 2.5th quantiles of the historically-observed **daily** log-returns of the FX spot rate, **using the whole data series**, following the steps as below:

1. Derive the closed-form formulas of the 97.5 and 2.5 quantiles of the log-returns of the GBM model.
2. Derive and as a function of these quantiles from the formulas in 1)
3. Calibrate and using the time series data and formula from 2)
4. In step 3) we have calibrated the parameters calibrated from daily log-returns. However, in the simulation step, we want to simulate the FX spot rate on a weekly simulation grid instead of daily. So, please calculate the corresponding and , from and derived in step 3), assuming 1 week = 5 days.

**Task 2: Simulation**

1. Test whether the daily log-returns of the underlying FX rate follows a normal distribution using any of the normality test methodologies and explain the testing results.
2. Simulate 1000 independent scenarios, with S(0) equal to the latest FX rate in the time series data, t=1week, 2week,……50weeks, and use parameters derived in calibration step 4). In this step we generate S(t) for each simulation path and at each simulation time point.
3. For each simulation path, at each simulation time point, insert the simulated value S(t) into the Black-Scholes formula for call option price (assuming no dividend payment):

Here assuming that **annualized** interest rate r=2%, strike K=S(0), maturity of the call option is 2 year = 104 weeks and use the parameters from **task 1, step 4**). In this step we get the re-evaluated call option value for all simulation path at each simulation time point.

1. At each simulation time point, calculate the mean and 97,5% quantile over all call option prices derived from each simulation paths in step 3), and plot the evolvements of the mean and 97.5% quantiles over time.

**Task 3: Back testing at the risk factor level**

The goal of the back testing at the risk factor level is to compare the model generated risk factor values against historically observed risk factor values, through which we could check whether the simulation model could properly reflect the historical movements of the risk factor. Please perform the back testing of the GBM model as simulation model for FX spot rate for EURUSD, following the steps as below:

1. Define 300 back testing windows with 1 week horizon. That is, for each back testing window, we predict the FX spot rate value at t0+5 days using the data at t0 and our simulation model, and compare it with observed value at t0+5 days. Here t0=1-Sep-2011, 8-Sep-2011, 15-Sep-2011… and every 5 business days / 1 week until you have 300 of them.
2. Simulate 1000 scenarios of the FX spot rate from t0 to t0+5 days for each t0 defined in step 1). Here, S(t0) is the time series data value on date t0, and please use and that are calibrated to the **weekly log-returns** of the FX spot rates in the past three years (or last 156 weeks) until each t0.
3. Take the observed values at each t0+5 and calculate how many simulated scenarios at t0+5 are below or equal to this observed value, say M, then divide M by 1000. In total we get 300 numbers of “M/1000”.
4. Check whether these 300 numbers follow a uniform distribution over [0,1] using any uniformity test. *Hint: for instance, Anderson-Darling, ChiSquare, Kolmogorov–Smirnov etc.*
5. Try to explain what can be concluded regarding the simulation model performance from your results obtained in step 4). *Hint: in step 3), we estimated the empirical CDFs of observed values that fall onto the simulated distributions.*